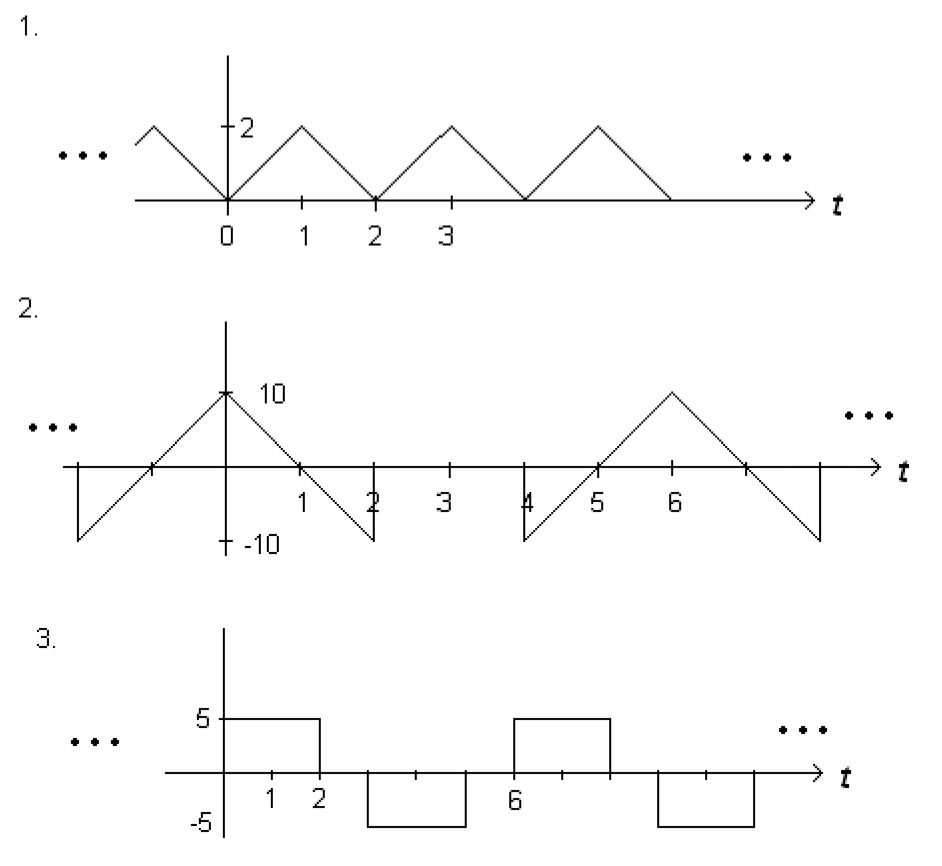
ECE 343 Jordan Dahl 4/4/2008

Find the Fourier series coefficients for the following waveforms. Use the derivative method if you like (remember to put back in the DC term if any).



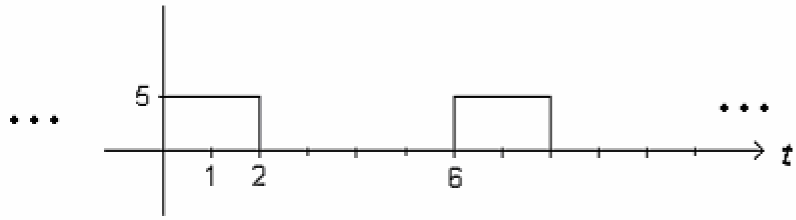
Find the coefficients for the following. Without integrating, you should be able to get the answer by inspection. Also, find the power in each signal by looking at the coefficients.

Note: since $T_o = 1/f_o$, $k/T_o = kf_o$. This means that k represents the multiple of the fundamental frequency. The fundamental is k=1. A fourth harmonic, for example, is k=4. DC is k=0.

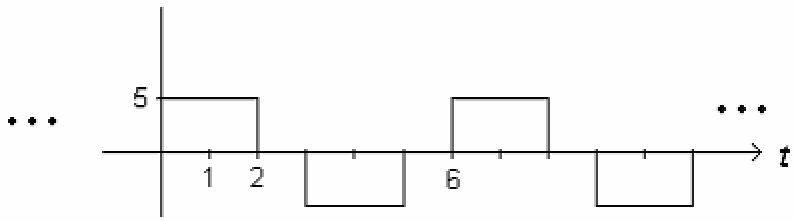
4. cos(t)
5. cos(t) + 5 cos(2t)
6. sin(5t) + 4 cos(3t)
7. j sin(5t) + cos(3t)
8. cos(15t) + cos(3t-10)

Find the coefficients for the following using the Poisson summation method.

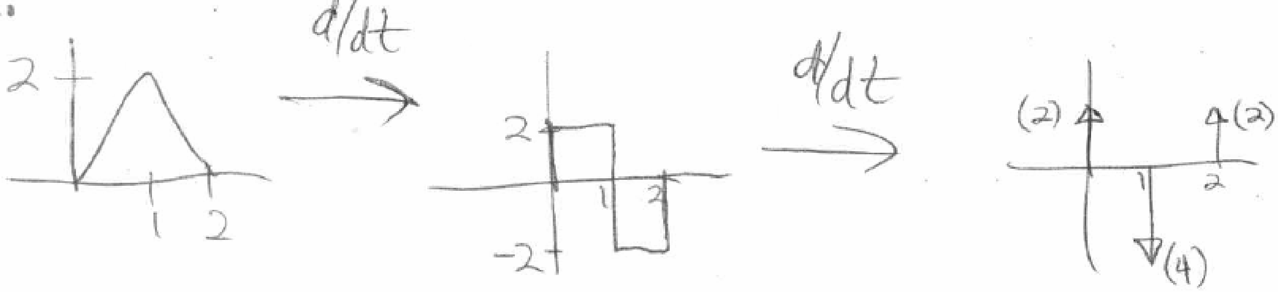
9.



10.



1.



we have $\frac{1}{T_0}\left[2 - 4e^{-j2\pi\frac{k}{T_0}(1)} + 2e^{-j2\pi\frac{k}{T_0}(2)}\right]$

$T_0 = 2. \Rightarrow \frac{1}{2}\left[2 - 4e^{-j\pi k} + 2e^{-j2\pi k}\right]$

to reverse the derivatives, divide by $\left(j2\pi\frac{k}{T_0}\right)^2$

$$\Rightarrow \frac{1}{2\left(j2\pi\frac{k}{2}\right)^2}\left[2 - 4e^{-j\pi k} + 2e^{-j2\pi k}\right] = \frac{-1}{2\pi^2k^2}\left[2 - 4e^{-j\pi k} + 2e^{-j2\pi k}\right]$$

(we could leave like this)

now notice the $e^{-j\pi k}$ and $e^{-j2\pi k}$:

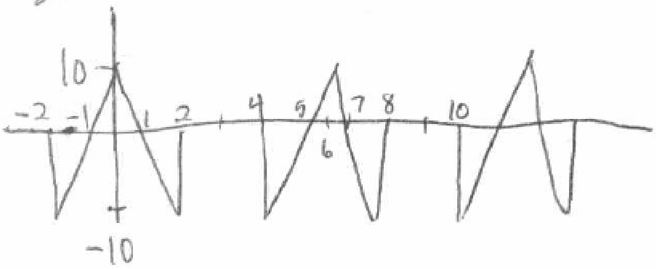
k	$e^{-j\pi k}$	$e^{-j2\pi k}$
1	-1	1
2	1	1
3	-1	1
4	1	1
⋮	⋮	⋮

$\Rightarrow$ so, $\frac{-1}{\pi^2k^2}\left[2-4+2\right]$ for k even

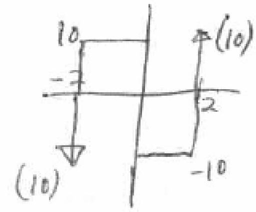
$\frac{-1}{\pi^2k^2}\left[2+4+2\right]$ for k odd

$$\Rightarrow \boxed{X[k] = \begin{cases} 0 & \text{for } k \text{ even} \\ \frac{-4}{\pi^2k^2} & \text{for } k \text{ odd} \\ 1 & \text{for } k=0 \end{cases}}$$

2.



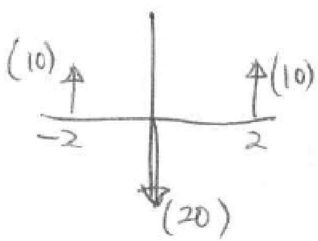
$\xrightarrow{d/dt}$



take these δ's before going further:

$$\frac{10}{T_0}\left[e^{-j2\pi\frac{k}{T_0}(2)} - e^{-j2\pi\frac{k}{T_0}(-2)}\right] \Big/ \left(j2\pi\frac{k}{T_0}\right)$$

next: d/dt →



$$\frac{10}{T_0}\left[e^{-j2\pi\frac{k}{T_0}(-2)} + e^{-j2\pi\frac{k}{T_0}(2)} - 2\right] \Big/ \left(j2\pi\frac{k}{T_0}\right)^2$$

$$\Rightarrow X[k] = \frac{-10}{T_0\, j2\pi\frac{k}{T_0}}\, j2\sin\left(\frac{4\pi k}{T_0}\right) + \frac{-10}{T_0\, 4\pi^2k^2/T_0^2}\left[-2 + 2\cos\left(\frac{4\pi k}{T_0}\right)\right]$$

$$T_0 = 6. \Rightarrow \boxed{X[k] = \frac{-10}{\pi k}\sin\left(\frac{\pi k 2}{3}\right) - \frac{15}{\pi^2k^2}\left[-2 + 2\cos\left(\frac{2\pi k}{3}\right)\right]}$$

3.

$T_0 = 6$

$$\Rightarrow \frac{5}{T_0}\left[e^{-j2\pi\frac{k}{T_0}(1)}\left(e^{+j2\pi\frac{k}{T_0}} - e^{-j2\pi\frac{k}{T_0}(1)}\right) - e^{-j2\pi\frac{k}{T_0}(4)}\left(e^{+j2\pi\frac{k}{T_0}} - e^{-j2\pi\frac{k}{T_0}}\right)\right]$$

$$= \frac{5}{6}\left[e^{-j\pi k/3}\, j2\sin\left(\frac{\pi k}{3}\right) - e^{-j4\pi k/3}\, j2\sin\left(\frac{\pi k}{3}\right)\right]$$

$$= j\frac{5}{3}\sin\left(\frac{\pi k}{3}\right)\left[e^{-j\pi k/3} - e^{-j4\pi k/3}\right]$$

now divide by $\left(j2\pi\frac{k}{T_0}\right)$

$$\rightarrow \frac{j\,5/3\,\sin\left(\frac{\pi k}{3}\right)}{j2\pi k/6}\left[e^{-j\pi k/3} - e^{-j4\pi k/3}\right] = \boxed{\frac{5\sin\left(\frac{\pi k}{3}\right)}{\pi k}\left[e^{-j\pi k/3} - e^{-j4\pi k/3}\right]}$$

(DC is 0)

---

4. $\cos(t) = \dfrac{e^{jt} + e^{-jt}}{2} = \frac{1}{2}e^{jt} + \frac{1}{2}e^{-jt}$

$$\Rightarrow \boxed{X[k] = \begin{cases} \frac{1}{2} & \text{for } k = \pm 1 \\ 0 & \text{elsewhere} \end{cases}}$$

$P = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \boxed{\frac{1}{2}}$

$\left(\Rightarrow \text{rms} = \sqrt{\left(\frac{1}{2}\right)} = \frac{1}{\sqrt{2}} \checkmark\right)$

5. $\cos(t) + 5\cos(2t) \Rightarrow \omega_0 = 1 \Rightarrow \cos(2t)$ is $k = 2$

$$\boxed{X[k] = \begin{cases} \frac{1}{2} & k = \pm 1 \\ \frac{5}{2} & k = \pm 2 \\ 0 & \text{otherwise} \end{cases}}$$

$P = 2\left(\frac{1}{2}\right)^2 + 2\left(\frac{5}{2}\right)^2 = \boxed{13}$

6. $\sin(5t) + 4\cos(3t)$

prime factor 5 and 3: $5 = 1 \cdot 5$, $3 = 1 \cdot 3$

The greatest common factor is 1. $\Rightarrow \omega_0 = 1$.

$\Rightarrow$ we have the 3^{rd} and 5^{th} harmonics

$$X[k] = \begin{cases} \frac{1}{j2} & k=5 \\ \frac{-1}{j2} & k=-5 \\ 2 & k=\pm 3 \\ 0 & \text{otherwise} \end{cases}$$

$$P = 2\left(\frac{1}{2}\right)^2 + 2(2)^2 = \boxed{8.5}$$

7. $j\sin(5t) + \cos(3t)$

same as last one

$$X[k] = \begin{cases} \frac{1}{2} & k=5 \\ \frac{-1}{2} & k=-5 \\ \frac{1}{2} & k=\pm 3 \\ 0 & \text{otherwise} \end{cases}$$

$$P = 4\left(\frac{1}{2}\right)^2 = \boxed{1}$$

8. $\cos(15t) + \cos(3t - 10)$ GCF of 3 and 15 is 3

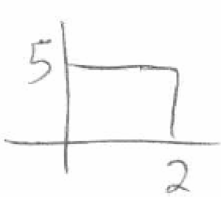
$\Rightarrow$ have fundamental and 5^{th} harmonic.

$$\cos(3t-10) = \frac{1}{2} e^{j(3t-10)} + \frac{1}{2} e^{-j(3t-10)} = \frac{1}{2} e^{-j10} e^{j3t} + \frac{1}{2} e^{j10} e^{-j3t}$$

$$\Rightarrow \quad X[k] = \begin{cases} \frac{1}{2} & k = \pm 5 \\ \frac{1}{2} e^{-j10} & k = 1 \\ \frac{1}{2} e^{j10} & k = -1 \\ 0 & \text{elsewhere} \end{cases}$$

$$P = 2\left(\frac{1}{2}\right)^2 + \left|\frac{1}{2} e^{-j10}\right|^2 + \left|\frac{1}{2} e^{j10}\right|^2$$

$$= 4\left(\frac{1}{2}\right)^2 = \boxed{1}$$

9. 

find Fourier transform first:

$$5\,\mathrm{rect}\left(\frac{t-1}{2}\right) \xrightarrow{\mathcal{F}} 5 \cdot 2\,\mathrm{sinc}(2f)\, e^{-j2\pi f(1)}$$

$$= 10\,\mathrm{sinc}(2f)\, e^{-j2\pi f} = X(f)$$

$$X[k] = \frac{1}{T_0} X\left(\frac{k}{T_0}\right) = \frac{1}{6}\, 10\,\mathrm{sinc}\left(2\frac{k}{6}\right) e^{-j2\pi\frac{k}{6}} \quad \left(\text{plug in } \frac{k}{6} \text{ for } f\right)$$

$$= \boxed{\frac{5}{3}\,\mathrm{sinc}\left(\frac{k}{3}\right) e^{-j\pi k/3} = X[k]}$$

10. $5\,\text{rect}\left(\frac{t-1}{2}\right) - 5\,\text{rect}\left(\frac{t-4}{2}\right)$

$$\xrightarrow{\mathcal{F}} 10\,\text{sinc}(2f)\,e^{-j2\pi f} - 10\,\text{sinc}(2f)\,e^{-j2\pi f(4)} = X(f)$$

$$\Rightarrow X[k] = \frac{1}{6}\,10\,\text{sinc}\left(2\frac{k}{6}\right)\left(e^{-j2\pi\frac{k}{6}} - e^{-j8\pi\frac{k}{6}}\right)$$

$$= \boxed{\frac{5}{3}\,\text{sinc}\left(\frac{k}{3}\right)\left(e^{-j\pi k/3} - e^{-j4\pi k/3}\right) = X[k]}$$

Also, we could have done:

$$x_{10}(t) = x_9(t) - x_9(t-3)$$

$$\Rightarrow X_{10}[k] = X_9[k] - X_9[k]\,e^{-j2\pi\frac{k}{T_0}(3)}$$

$$= \frac{5}{3}\,\text{sinc}\left(\frac{k}{3}\right)e^{-j\pi k/3}\left(1 - e^{-j\pi k}\right)$$

$$= \frac{5}{3}\,\text{sinc}\left(\frac{k}{3}\right)\left(e^{-j\pi k/3} - e^{-j4\pi k/3}\right) \qquad \text{(same result)}$$

This is the same result as #3.

If you know transforms, this method is easy. If all you know is $\delta(t) \longrightarrow \frac{1}{T_0}$, the derivative method is easy. The more transforms you know, the less tedious algebra you need to do.

Matlab code to check our answers:
Save this to a .m file in Matlab's working directory and run it.

```
clear
% #1.
t=0:.001:4;
T=2;
k=-20:1:20; %can get better approximation with more k's
i=find(k~=0);
Xk(i)=(-4)./(pi^2*k(i).^2); %do all X[k] except at k=0
%make zero if k is even:
Xk=Xk.*mod(k,2);
%add k=0 term:
Xk(k==0)=1;
x=(Xk*exp(j*2*pi*k'*t./T));
plot(t,x)
title('1.')

pause
% hit a keyboard button to resume

% #2.
t=0:.001:8;
T=6;
Xk=-10*sin(pi*k*2/3)./(pi*k)-15*(-2+2*cos(2*pi*k/3))./(pi*k).^2;
Xk(k==0)=0;
x=(Xk*exp(j*2*pi*k'*t./T));
plot(t,x)
title('2.')

pause

% #3.
k=-40:1:40;
T=6;
Xk=5*sin(pi*k/3).*(exp(-j*pi*k/3)-exp(-j*pi*4*k/3))./(pi*k);
Xk(k==0)=0;
x=(Xk*exp(j*2*pi*k'*t./T));
plot(t,x)
title('3.')

pause

% #4.
k=[-1 1];
T=2*pi;
Xk=[.5 .5];
x=(Xk*exp(j*2*pi*k'*t./T));
g=cos(t);
plot(t,x,t,g)
title('4.')

pause

% #5.
k=[-1 1 -2 2];
T=2*pi;
Xk=[.5 .5 2.5 2.5];
x=(Xk*exp(j*2*pi*k'*t./T));
g=cos(t)+5*cos(2*t);
plot(t,x,t,g)
title('5.')

pause
```

```
% #6.
k=[-3 3 -5 5];
T=2*pi;
Xk=[2 2 -1/(j*2) 1/(j*2)];
x=(Xk*exp(j*2*pi*k'*t./T));
g=sin(5*t)+4*cos(3*t);
plot(t,x,t,g)
title('6.')

pause

% #7.
k=[-3 3 -5 5];
T=2*pi;
Xk=[.5 .5 -1/(2) 1/(2)];
x=(Xk*exp(j*2*pi*k'*t./T));
g=j*sin(5*t)+cos(3*t);
plot(t,x,t,g)
title('7.')

pause

% #8.
k=[-1 1 -5 5];
T=2*pi/3;
Xk=[.5*exp(j*10) .5*exp(-j*10) .5 .5];
x=(Xk*exp(j*2*pi*k'*t./T));
g=cos(15*t)+cos(3*t-10);
plot(t,x,t,g)
title('8.')

pause

% #9.
k=-20:1:20;
T=6;
Xk=(5/3)*sin(pi*k/3).*exp(-j*pi*k/3)./(pi*k/3);
Xk(k==0)=5/3;
x=(Xk*exp(j*2*pi*k'*t./T));
plot(t,x)
title('9.')

pause

% #10.
k=-40:1:40;
T=6;
Xk=(5/3)*sin(pi*k/3).*(exp(-j*pi*k/3)-exp(-j*4*pi*k/3))./(pi*k/3);
Xk(k==0)=0;
x=(Xk*exp(j*2*pi*k'*t./T));
plot(t,x)
title('10.')
```